Anomalous Higgs Interactions in Gauge-Higgs Unification

Seminar @ Osaka Univ.

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Based on

- "Anomalous Higgs Interactions in Gauge-Higgs Unification",
  w./ K. Hasegawa (Harish Chandra Inst.), N. Kurahashi and K. Tanabe (Kobe Univ.), 1201.5001 [hep-ph]
- Anomalous Gauge Interactions in Gauge-Higgs Unification,
  a paper in preparation

I. Introduction

The standard model has unsettled problems in its Higgs sector:

(1) The hierarchy problem (how to maintain $M_W \ll \Lambda$ naturally?) : Higgs mass gets "quadratic divergence" Λ^2

(2) The origin of hierarchical fermion masses and flavor mixings ?

(3) The origin of CP violation still seems to be not conclusive yet.

(4) The origin of Higgs itself.

$\leftarrow$ there is no guiding principle (symmetry) to restrict the interactions of Higgs

We discuss GHU as a scenario of New Physics, which is expected to shed some lights on these problems relying on higher dimensional gauge symmetry.

"<u>Gauge-Higgs unification (GHU)</u>" scenario (Manton, Hosotani)

unification of gauge (s=1) & Higgs (s=0) interactions

: realized in higher dimensional gauge theory

$$A_M = (A_\mu, A_y) \quad (5D), \quad A_y^{(0)}(x) = H(x) : \ \text{Higgs}$$

Higgs potential is radiatively induced and its VEV realizes dynamical breaking of gauge symmetry: "Hosotani mechanism"

Y. Hosotani (Phys. Lett. B126 ('83) 309)

The quantum correction to m_H is finite because of the higher dimensional gauge symmetry, once all KK modes are summed up (w./ H. Hatanaka , T. Inami, Mod. P. L. A13('98)2601)

→ A new avenue to solve the hierarchy problem without invoking SUSY

The minimal model

In GHU, gauge group should be enlarged.
SU(3) electro-weak GHU model has been constructed (M. Kubo, C.S. L. and H. Yamashita, Mod. P. L. 17('02)2249; C. A. Scrucca, M. Serone and L. Silvestrini, N. P. B **669**, 128 (2003).)

SU(3) $\to$ SU(2)$_L$ x U(1)$_Y$ by orbifolding S^1/Z_2 (Y. Kawamura):

$$Z_2: \quad y \to -y$$

For quarks (SU(3) triplet), $\Psi(-y) = \mathcal{P}\gamma^5\Psi(y) \quad (\mathcal{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix})$

KK Zero-modes of Gauge-Higgs sector :

$$A_\mu^{(0)} = \frac{1}{2}\left(\begin{array}{cc|c} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ \hline 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{array}\right)$$

$$A_y^{(0)} = \frac{1}{\sqrt{2}}\left(\begin{array}{cc|c} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \hline \phi^- & \phi^{0*} & 0 \end{array}\right)$$

Higgs doublet

$$8 \leftarrow 3 + 1 + 2 \times 2$$

Unification of Gauge-Higgs sector in SU(3) adjoint !

However, $\sin^2\theta_W = \frac{3}{4}$ (at the SU(3) unitifaction)

To remedy this, G_2 may be helpful (Csaki, Grojean & Murayama)

(N.B.) Closely related with other ideas of new physics:

- Close relation to "Little Higgs"

The circumstantial evidence:

(i) In both, gauge group of SM is enlarged to group G, and Higgs is identified with NG boson of G/H in Little Higgs , and A_y of G/H in GHU.

(ii) Both have sift symmetries,

$$A_y \to A_y + \partial_y \lambda, \quad G \to G + \text{const.}$$

Little Higgs $\Leftrightarrow$ Dimensional Deconstruction $\Leftrightarrow$ GHU

(Or through AdS-CFT)

- Close relation to Superstring

The (bosonic part of) point particle limit of open superstring theory, 10D SUSY Y.-M. theory, is a sort of GHU.

<u>SUSY YM theory is possible only for D = 3, 4, 6 & 10</u>

(D−2 = 1,2,4 & 8)

$$D - 2 = r\ 2^{[\frac{D-2}{2}]} \quad (r = \frac{1}{2} \text{ for Majorana-Weyl})$$

We have no scalar field: (A_M, ψ)

$\rightarrow$ Higgs stems from A_M

In the breaking $E_8 \rightarrow E_6$

27 repr. of E_6 (16 + 10 + 1 of SO(10)) stems from the adjoint repr. of E_8

II. Anomalous interactions as characteristic prediction of GHU (w./ K. Hasegawa, N. Kurahashi and K. Tanabe, 1201.5001 [hep-ph])

In gauge theories with SSB fermion mass term is written as

$$m(v)\bar{\psi}\psi$$

where $m(v)$ is a function of the VEV $v = \langle H \rangle$.
The interaction of physical Higgs field h with fermion is expected to be provided by

$$v \quad \rightarrow \quad v + h$$

For instance, in the SM

$$m(v) = fv \ (f : \text{Yukawa coupling})$$

and the Yukawa interaction of h with ψ is given as

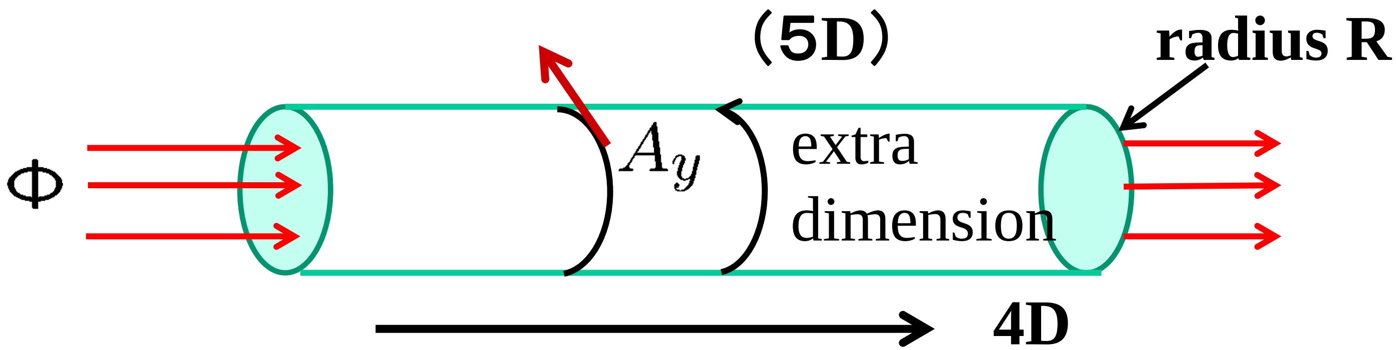
$$m(v+h)\bar{\psi}\psi = f(v+h)\bar{\psi}\psi,$$

$$f = \frac{dm(v)}{dv}$$

In GHU, $H(\leftarrow A_y^{(0)})$ has a physical meaning as
Wilson loop (AB phase):

$$W = P\ e^{i\frac{g}{2}\oint A_y dy} = e^{ig_4 \pi R A_y^{(0)}}$$

Circle : non-simply-connected



$$e^{ie\oint A_y dy} = e^{ie 2\pi R A_y} = e^{ie\Phi} \quad \Rightarrow \quad A_y = \frac{\Phi}{2\pi R}$$

↑
Wilson - loop

(Abelian case)

Thus we expect physical observables have periodicity in H

$$v \quad \to \quad v + \frac{2}{g_4 R} \quad (g_4 : \text{4D gauge coupling})$$

, just as what happens in the quantization condition of magnetic flux in super-conductor :

$$\Phi = \frac{2\pi}{e} n \ (n : \text{integer})$$

(N.B.) ・Effective potential of v is a typical example:

$$V(v) \propto \frac{3}{4\pi^2} \frac{1}{(2\pi R)^4} \sum_{n=1}^{\infty} \frac{\cos(n g_4 \pi R v)}{n^5}$$

(Poisson re-summation) n: "winding number"

$$\cos(n g_4 \pi R v) = \mathrm{Re}(W^n)$$

・ In the simply-connected S^2, Higgs mass vanishes (w./ N. Maru, K. Hasegawa, J.Phys.Soc.Jap. 77 (08)074101 :hep-th/0605180)

Also for fermions masses, we will find for light quarks,

$$m(v) \propto \sin(\frac{g_4}{2}\pi R v)$$

$$\Downarrow$$

$$m(v+h) \propto \sin(\frac{g_4}{2}\pi R(v+h))$$: non-linear in h !

$$\Downarrow$$

$$f = \frac{dm(v)}{dv} \propto \cos(\frac{g_4}{2}\pi R v)$$

: even vanishes for $x \equiv \frac{g_4}{2}\pi R v = \frac{\pi}{2}$!

This kind of anomalous Higgs interaction has been pointed out (in R-S 5D space-time and for SO(5) x U(1) model) by

Y. Hosotani, K. Oda, T.Ohnuma, Y. Sakamura, P.R.D78('08)096002;
Y. Hosotani and Y. Kobayashi, P.L. B674('09)192

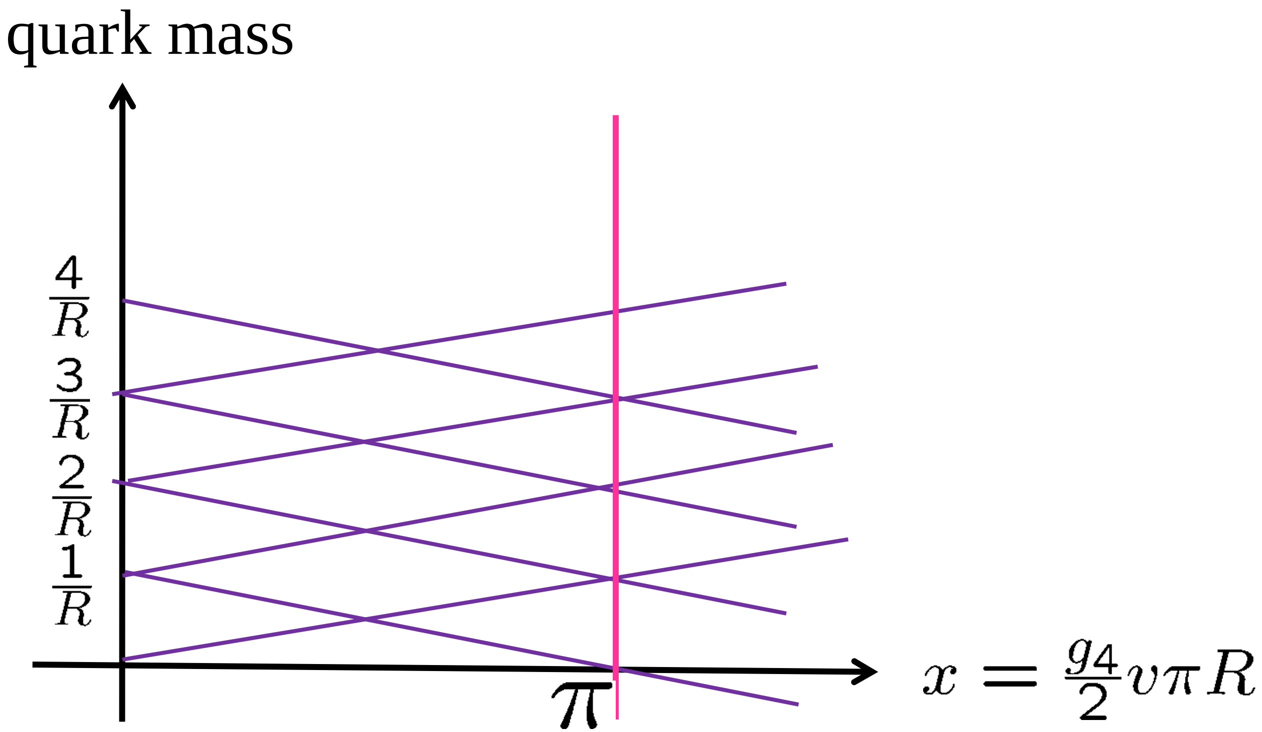
$\Rightarrow$ Higgs may be dark matter !

However, the Yukawa coupling should be linear in h as in SM:

After the replacement $v \rightarrow v + h$ free lagrangian is given as

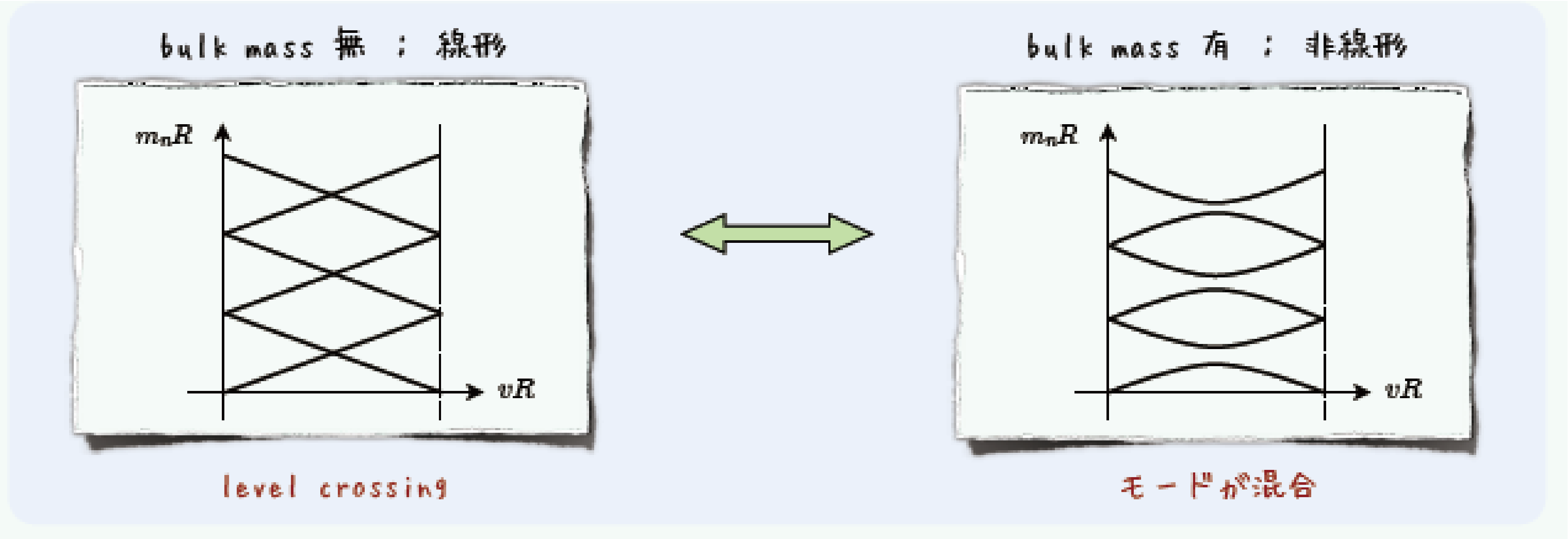
$$\bar{\psi}\{i\partial_\mu\gamma^\mu - \gamma_5\partial_y + i\gamma_5 g_4\frac{\lambda_6}{2}(v+h) - M\epsilon(y)\}\psi$$

For M = 0 (heavy quark) m(v) is linear in v:



(N.B.)

The bulk mass M breaks translational invariance in the extra space and causes mixings between different KK modes:



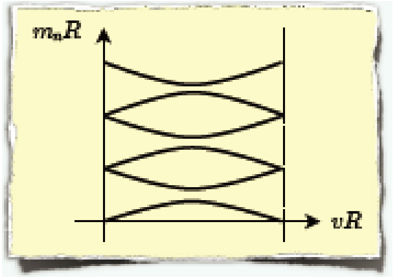
At the first glance there seems to be contradiction.
(Our purpose)
- To understand how these two "pictures" ("non-linear or linear") are reconciled each another in simple setting: SU(3) model on flat $M^4 \times S^1/Z_2$

<u>The minimal SU(3) (×SU(3)$_c$) model</u> on $M^4 \times S^1/Z_2$

The mass eigenvalues for d quark (1 generation only) are given by

$$\frac{m_n^{(\pm)}}{\sqrt{m_n^{(\pm)2}-M^2}}\sin(\sqrt{m_n^{(\pm)2}-M^2}\pi R) = \pm(-1)^n \sin(\tfrac{g_4}{2}v\pi R)$$

periodicity



For light quarks $m_0^{(+)} \ll M$, zero-mode mass is approximated as

$$m_0^{(+)} \simeq \frac{M}{\sinh(\pi MR)}\sin(\tfrac{g_4}{2}v\pi R) \propto \sin(\tfrac{g_4}{2}v\pi R)$$

: trigonometric & exponential suppression

How to reconcile two pictures ?

After the replacement $v \rightarrow v + h$, the operators of 4D mass & Yukawa coupling

$$\int_{-\pi R}^{\pi R} \bar{\psi}\{-\gamma_5 \partial_y + i\gamma_5 g_4 \frac{\lambda_6}{2}(v + h) - M\epsilon(y)\}\psi \ dy$$

is written in a matrix form

$$M_m + hM_Y$$: linear in h

in the base of physical quark states (including KK modes)

$$\psi_{L,R}^{(n)}(x) \ (n = 0, 1, \ldots)$$

where

$M_m = \mathrm{diag}(m_0, m_1, m_2, \ldots)$: diagonal mass matrix

M_Y : "Yukawa coupling matrix"

(N.B.)

- $m(v+h)$ is the eigenvalue for the whole matrix $M_m + hM_Y$, which may be non-linear in h, in general. $\rightarrow$ no contradiction
- In the case $m(v+h)$ is non-linear, M_Y should be off-diagonal, since otherwise the eigenvalues would be linear in h.

(wisdom in perturbation theory of quantum mechanics)

At 1-st order of perturbation H', the energy shift is given by $\langle n|H'|n\rangle$

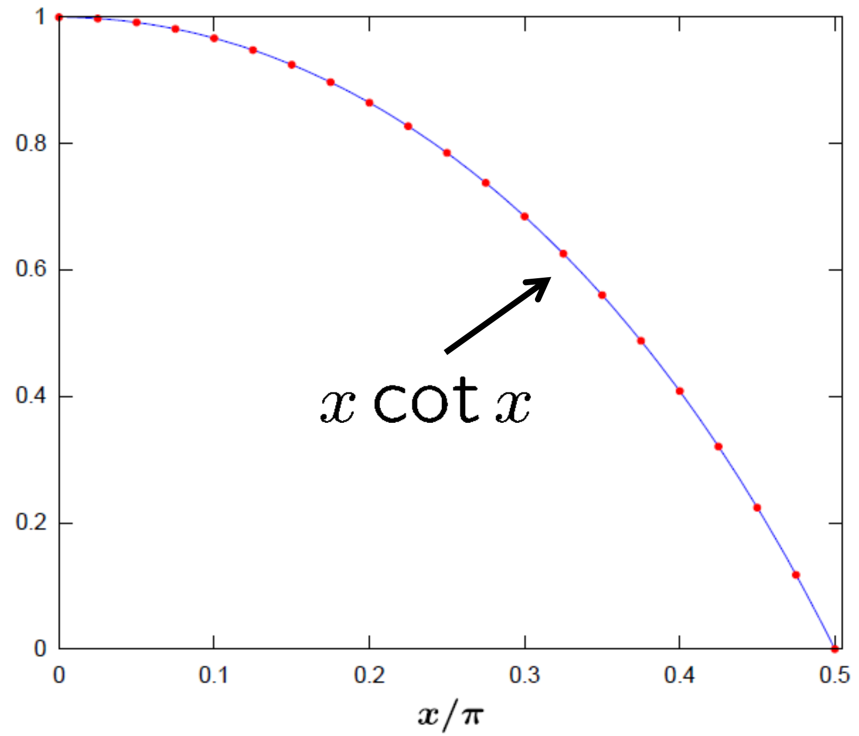
$\Rightarrow$ Treating hM_Y as a perturbation, $\frac{dm(v)}{dv} = m'(v)$

should be given by the diagonal element of M_Y : $(M_Y)_{nn}$

We calculated the both of $m'(v)$ and $(M_Y)_{nn}$ to confirm they just coincide.
Especially, when $x \equiv \frac{g_4}{2}\pi R v = \frac{\pi}{2}$, M_Y is completely off-diagonal.

The anomalous Yukawa coupling with zero-mode light quark, $f_{GHU} = (M_Y)_{00}$, is well approximated by the formula,

$$\frac{f_{GHU}}{f_{SM}} \simeq x \cot x$$



(N.B.)

- In the "decoupling limit"

$$x = \frac{g_4}{2} v\pi R \ll 1 \leftrightarrow M_W \ll \frac{1}{R}$$

SM prediction is recovered.

- vanishes at $x = \frac{\pi}{2}$

Thus, as far as we retain in the 0-mode sector, both approaches give identical result, as far as linear interaction of h is concerned.

Then what about the quadratic Higgs interaction with zero-mode quark, $\bar{\psi}^{(0)}\psi^{(0)}h^2$?

quantum mechanics (2nd order perturbation) tells us:

$$-\sum_{m\neq n}\frac{|\langle m|H'|n\rangle|^2}{E_m-E_n} \quad \Rightarrow \quad m''(v) = -\sum_{n\neq 0}\frac{|(M_Y)_{n0}|^2}{m_n-m_0}$$

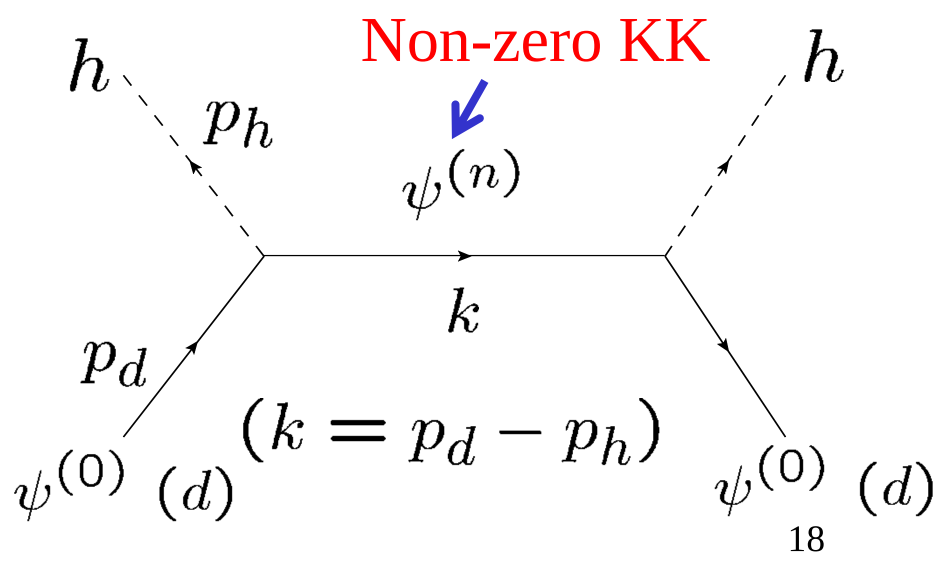
On the other hand, direct calculation of Feynman diagram by use of off-diagonal Yukawa couplings $(M_Y)_{n0}$ provides

$$\sum_{n\neq 0}\frac{|(M_Y)_{n0}|^2}{k_\mu\gamma^\mu-m_n}$$

In the limit $p_h \to 0$

, by use of $(p_d)_\mu\gamma^\mu = m_0$

both coincide



(N.B.)

- This is a reasonable result, since $m(v+h)$ is obtained regarding h as a constant field ($P_h = 0$).
- On the other hand, when Higgs mass and/or P_h become comparable with the compactification scale 1/R, as may be the case in the experiments at LHC and linear collider, two pictures provide different predictions.

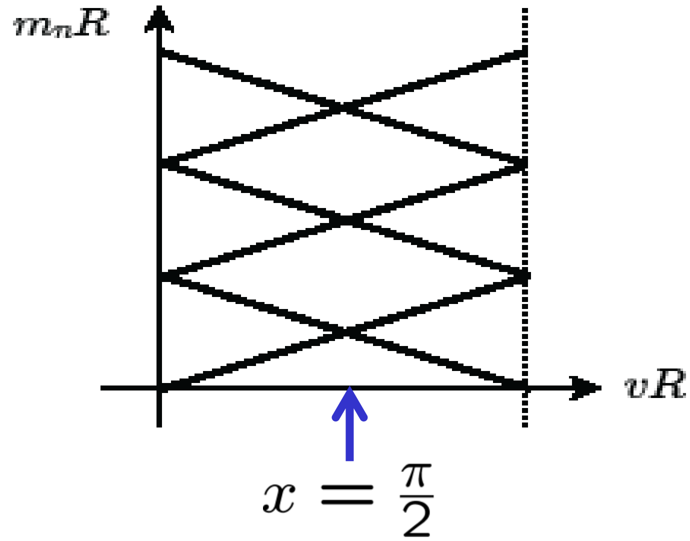
What about Higgs interaction with massive gauge bosons ?

The coupling of $W_\mu^+ W^{-\mu} h$ is "almost normal", as the mass spectrum is linear in v , in contrast to the case of R-S background (Hosotani & Sakamura, P. T. P. 118, 935 (2007)):

$ig_4 M_W \quad \text{for} \quad x < \frac{\pi}{2}$

$-ig_4 M_W \quad \text{for} \quad x > \frac{\pi}{2}$

singular at $x = \frac{\pi}{2}$



Mixing between different KK modes via fermion loop is expected at $x = \frac{\pi}{2}$, leading to vanishing Higgs coupling there, as suggested by H-parity argument.

(N.B.)

On the R-S background, translational invariance is "always" broken by the warp factor $e^{-\kappa|y|}$

$\rightarrow$ trigonometric $m^2(v)$ $\rightarrow$ anomalous Higgs interactions

Anomalous gauge interactions with fermion

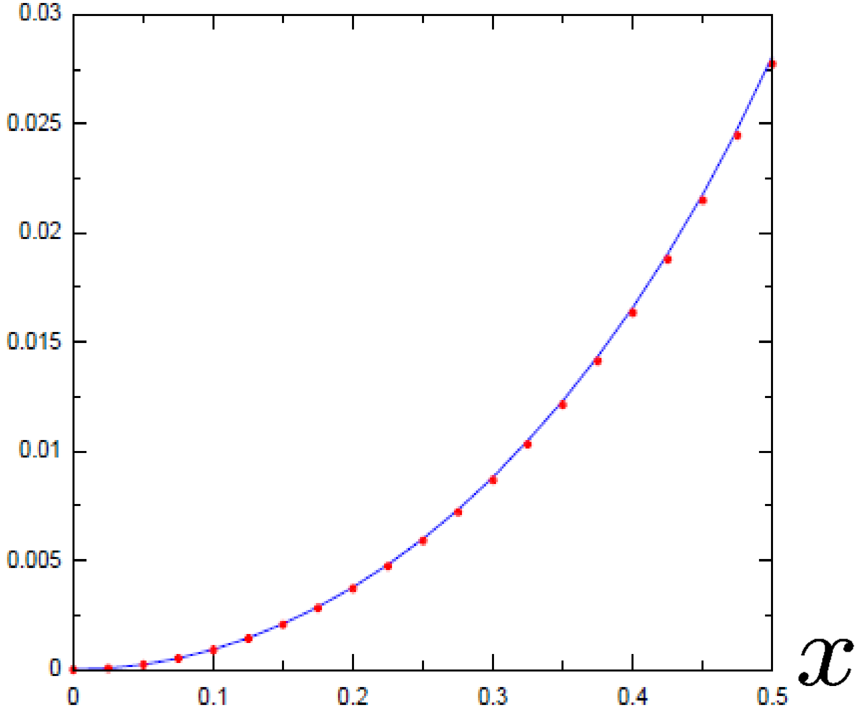
We also have found the gauge interactions of zero mode gauge bosons, which acquire masses by SSB, $W^{\pm}$, Z^0 , with fermions are anomalous .

For lighter quarks, the deviation from SM is well approximated by

$$\frac{g_W}{g_{SM}} \simeq 1 - \frac{x^2}{(2\pi RM)^2 + x^2}$$ (M: fermion bulk mass)

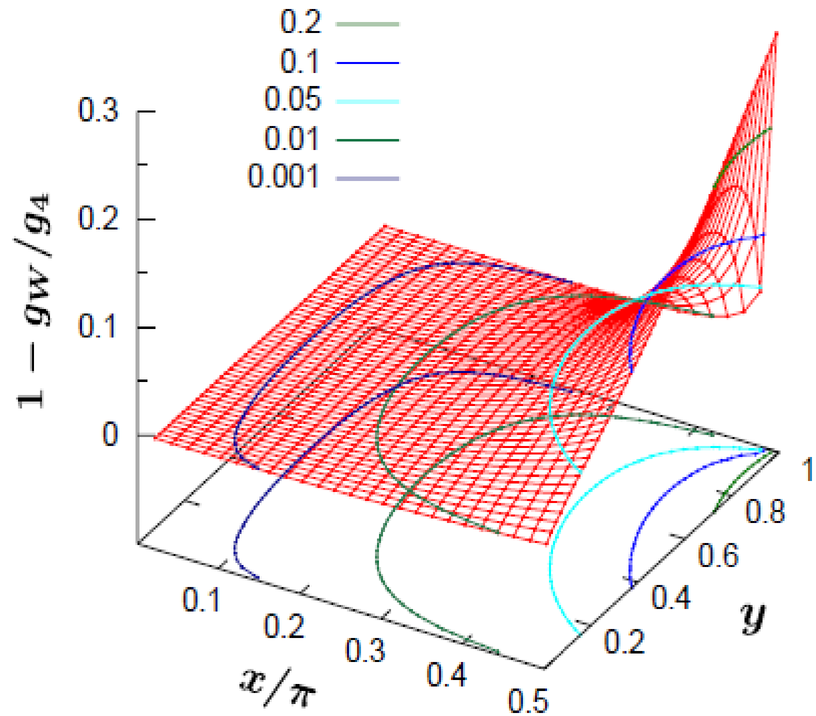
- The deviation is large for larger x.
  ← zero-mode is replaced by 1st KK mode, whose interaction differs from that of SM
- The replacement does not happen for photon or gluon.

$1 - \frac{g_W}{g_{SM}}$ (a) bottom quark



$y = \frac{m_b}{m_W} \approx 5.81 \times 10^{-2}$

(b) general case



The blue line in (a) stands for the approximated "shift" in (6.33). The possible largest value in this figure is about 0.29 ($g_W \sim 0.71 g_4$).

H parity

At the extreme case, $x = \frac{\pi}{2}$ (diagonal) Yukawa coupling vanishes and Higgs becomes rather stable $\Rightarrow$ dark matter ?

This suggests the presence of "parity" under which Higgs is only particle with odd eigenvalue among the SM particles

$\Rightarrow$ H parity

(Y. Hosotani, M. Tanaka, N. Uekusa, P.R. D82 (2010) 115024)

Question

- What symmetry in our model corresponds to H parity ?
- Is it meaningful ? It seems to be broken by the VEV of H :

$$\text{H parity: } H = v + h \quad \to \quad -H = -v - h$$

$$\text{H parity}: \quad \psi \ \rightarrow \ P\ \psi, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Rightarrow \qquad P\langle A_y\rangle P^{-1} \neq \langle A_y\rangle, \qquad \langle A_y\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{v}{\sqrt{2}} \\ 0 & \frac{v}{\sqrt{2}} & 0 \end{pmatrix}$$

However, Wilson-loop

$$W = e^{ig_4\pi R\langle A_y\rangle} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(2x) & i\sin(2x) \\ 0 & i\sin(2x) & \cos(2x) \end{pmatrix}$$

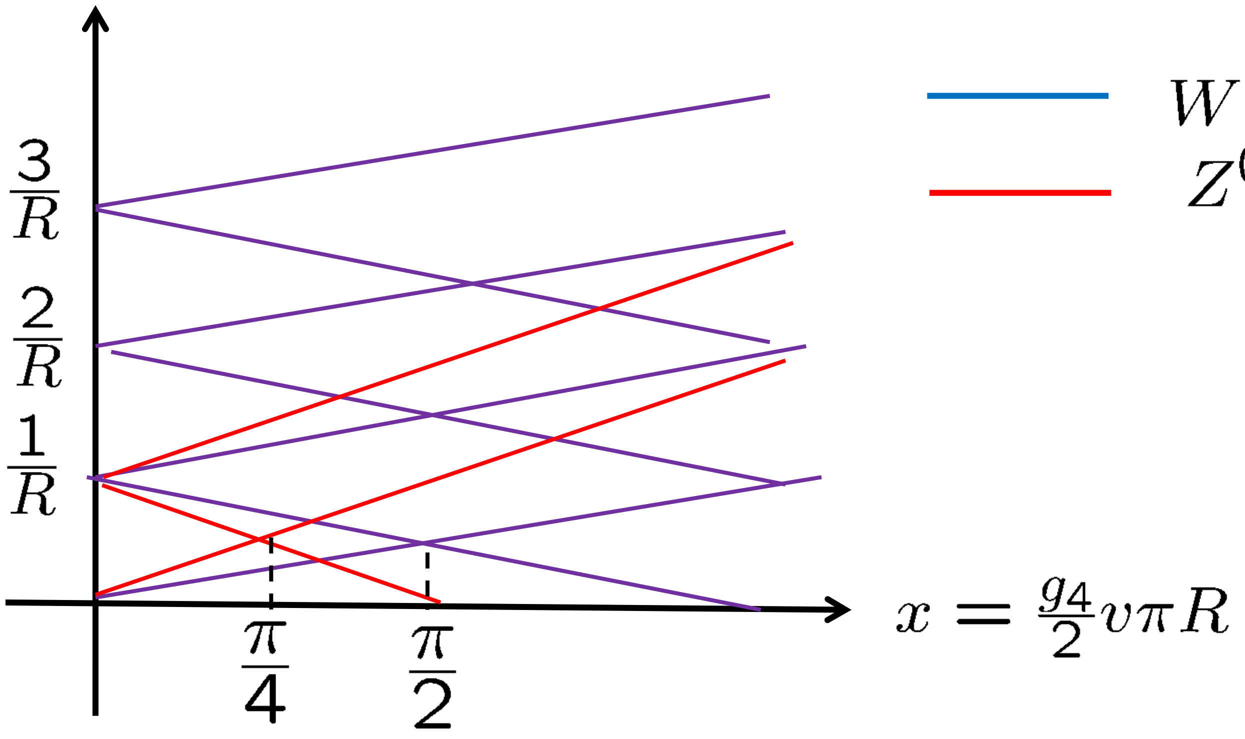
is invariant for $x = \frac{\pi}{2}$

(N.B.)

However, for $x = \frac{\pi}{2}$, SU(2) x U(1) $\Rightarrow$ U(1) x U(1)

(Kubo, L., Yamashita, ('02))

Gauge boson mass



(Comments on the “reality“ of the model)

- Is unitarity OK in the W scattering ?
  The coupling of Higgs with W is just as in the SM
- Does it pass the precision test ?
  Non-vanishing S-parameter at tree level ?
  Again, the zero-mode gauge boson sector is as usual.
  → S & T parameters vanish at tree level
  (w./ N.Maru, Phys.Rev. D75 (2007) 115011)
  Or, operator relevant for S and T,

$$S: \quad (\varphi^\dagger F_{\mu\nu}\varphi)B^{\mu\nu}, \quad T: \quad (\varphi^\dagger D_\mu \varphi)(\varphi^\dagger D^\mu \varphi)$$

does not exist at the tree level in our model.

(N.B.)

The operator may be realized, in general, once the mass spectrum of massive gauge bosons shows nonlinear behavior, like in the case of R-S background:

Replacing v by h, we may be able to get the higher (mass-)dimensional operators:

$$W_u^+ W^{+\mu} \sin^2\left(\frac{g_4}{2} v\pi R\right) \quad \rightarrow \quad W_u^+ W^{+\mu} \sin^2\left(\frac{g_4}{2} h\pi R\right)$$